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Albert-Ludwigs-Universität Freiburg
Sommersemester 2024

Topology

Problem Sheet 7

Deadline: 11 June 2024, 15h

Exercise 1 (6 Points)**.**
Let $f : X \to Y$ be a continuous map between topological spaces.

a) Show with an explicit example that the preimage of a compact subset K of Y need not be compact in X.

b) Assume now that f is closed and surjective. If U is an open subset containing the fiber $f^{-1}(\{y\})$ for some $y \in K$, show that $f(U)$ contains an open neighborhood of y.

c) Assume now that f is closed, surjective and that for every y in Y the fiber $f^{-1}(\{y\})$ is a compact subset of X. Show that for any compact subset K of Y the preimage $f^{-1}(K)$ is compact in X.

Exercise 2 (4 Points)**.**
Let X be the topological space $(\mathbb{C}, \mathcal{T}_{\text{cofin}})$ of the complex numbers equipped with the cofinite topology.

a) Is X sequentially compact?

b) Is X a limit-point compact space?

Exercise 3 (4 Points)**.** Consider the topology $\mathcal{T}$ on $\mathbb{R}$ generated by the half-rays $(a, +\infty)$ with a in $\mathbb{R}$.

a) Determine whether it is finer or coarser than the euclidean topology. Is it T_1?

b) Show that $(\mathbb{R}, \mathcal{T})$ is first-countable. Is it also second-countable?

c) Show that this topological space is limit-point compact.

d) Does the sequence $\{-n\}_{n\in\mathbb{N}}$ have a convergent subsequence?

Exercise 4 (6 Points)**.**
Let $(X, \mathcal{T})$ be a topological space.

a) If the path α and β are homotopic, then show that the path $\alpha \star \gamma$ and $\beta \star \gamma$ are also homotopic for any path γ such that $\gamma(0) = \alpha(1) = \beta(1)$.

b) Let $X = \mathbb{R}^2 \setminus \{(0, 0)\}$ be the *punctured plane* with the subspace topology from euclidean plane. Consider a loop α based at $x_0 = (1, 0)$ such that the x-coordinate of α is always non-negative. Show that α is homotopic to the constant loop C_{x_0}.

Die Übungsblätter können zu zweit eingereicht werden. Abgabe der Übungsblätter im entsprechenden Fach im Keller des mathematischen Instituts.